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Using Hybrid And LQR Method Control Of A Self-Erecting Rotary Inverted Pendulum System Based On PIC 18F4431

Huu Chan Thanh Nguyen[#1], An Wen Shen[#2]

[#]*Department of Control Science And Engineering, HuaZhong University Of Science And Technology*
1037 LuoYu Road, WuHan, HuBei, China

***Abstract*— The rotary inverted pendulum system was a highly nonlinear model, multivariable and absolutely unstable dynamic system. This paper presents hybrid control method for swing up and LQR control method for the stabilization. The flexibility of the hybrid control method based on the selection of heuristic swing up controller and energy swing up controller at the different positions of pendulum. The robustness of the LQR control method for the stabilization is tested by attaching a screw driver at the tip of pendulum. The proposed controllers for self-erecting rotary inverted pendulum system is simulated by using MATLAB-SIMULINK and implemented on an experiment system using PIC 18F4431 microcontroller.**

***Keywords*— LQR control, hybrid control, rotary inverted pendulum, self-erecting.**

I. INTRODUCTION

The Rotary Inverted Pendulum Systems (RIPS) is a challenging problem in the area of control systems. RIPS are nonlinear and open loop unstable. Therefore, they are excellent apparatus for testing different control schemes.

This paper focuses on analysis and design a hybrid controller for swing up a self-erecting RIPS. The hybrid controller is the combination of heuristic swing up controller and energy swing up controller. At the hanging down position of pendulum, using the energy swing up controller, that helps pendulum swing up with a single swing. And when the pendulum move from upward balance state to unstable state, using the heuristic swing up controller will bring the pendulum return upward balance state fast and smoothly.

Stabilization of RIPS at upward position using LQR control method is also proposed in this paper. LQR controller is robust and easily tuned in experiment system.

In recent year, 8 bit microcontroller was integrated more and more peripheral interface module as incremental encoder interface module, pulse width modulation module, universal asynchronous receiver transmitter module… This give us a flexible and cheap solution to control a system without using expensive digital signal processing IC.

The third contribution of this paper is the implementation of microcontroller PIC 18F4431 based design, help RIPS can be operated on a stand-alone basis with a low price controller.

II. SYSTEM SETUP AND DYNAMICS

A. System setup

The physical arrangement of RIPS is shown in Fig. 1a and the implement details on hardware PIC 18F4431 based design are provided in Fig. 2. The RIPS consists of a pendulum mounted on one end of a rotating arm. The other end of the arm is mounted on a Faulhaber direct current gear motor shaft. The arm rotates in the horizontal plane while the pendulum rotates in a plane that is always perpendicular to the rotating arm. Two incremental encoder with 1000 pulse per round are used to measure the angular position of the direct current motor θ and the pendulum position α.

Control system uses two PIC 18F4431 with master slaver structure, interface via UART channel. PIC master read the value of θ, request the value of α from slaver, algorithm processing and generate PWM signal supply to motor. PIC slaver read the value of α and sent to PIC master. The sampling time of 4 ms is chosen for PIC master.

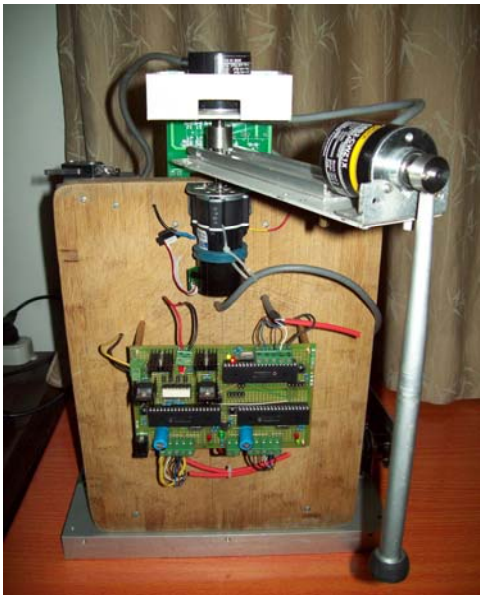


Fig 1. Proposed rotary inverted pendulum system

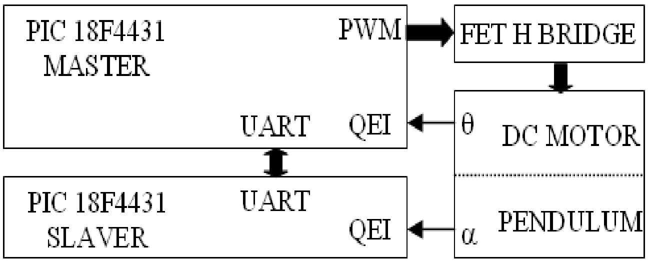


Fig 2. Hardware PIC 18F4431 based design

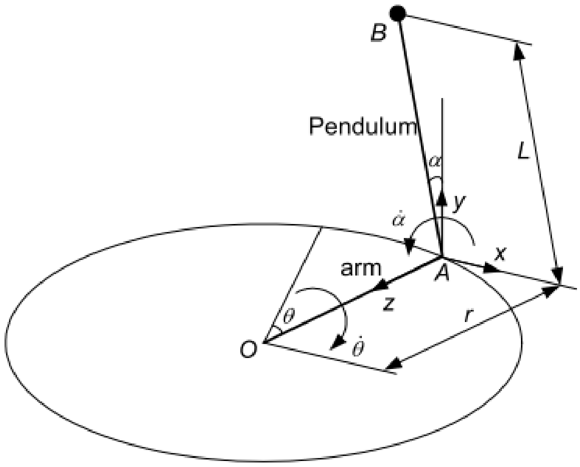


Fig 3. Simplifier model of the RIPS

B. RIPS Dynamics

The governing differential equations of the RIPS [1][3] [6] at Fig. 3 are as follows:

$$\begin{cases} a\ddot{\theta} - b.\cos\alpha.\ddot{\alpha} = c.V_m - d.\dot{\theta} - b.\sin\alpha.\dot{\alpha}^2 \\ -b\cos\alpha.\ddot{\theta} + e.\ddot{\alpha} = f.\sin\alpha \end{cases} \quad (1)$$

where:

$$\begin{cases} a = J_{eq} + mr^2 + \eta_g K_g^2 J_m \\ b = mrL;\ c = \dfrac{\eta_m \eta_g K_t K_g}{R_a}; \\ d = \dfrac{\eta_m \eta_g K_m K_t K_g^2}{R_a} + B_{eq} \\ e = J_B + mL^2;\ f = mgL \\ \alpha : \text{pendulum position} \\ \dot{\alpha} : \text{pendulum velocity} \\ \ddot{\alpha} : \text{pendulum acceleration} \\ \theta : \text{arm position} \\ \dot{\theta} : \text{arm velocity} \\ \ddot{\theta} : \text{arm acceleration} \end{cases} \quad (2)$$

Solving for $\ddot{\alpha}$ *and* $\ddot{\theta}$ from (1), result in the following nonlinear equations:

TABLE 1. PARAMETERS OF THE EXPERIMENTAL SYSTEM

Symbol	Description	Value	Unit
g	Gravity acceleration	9.81	m/s^2
B_{eq}	Equivalent viscous damping coefficient	0.004	Nm/(rad/s)
K_g	Motor gear ratio	64	
η_g	Gearbox efficiency	0.9	
K_t	Motor torque constant	0.0134	N-m/A
K_m	Back-emf constant	0.0134	V-s/rad
J_m	Moment of inertia of the rotor of the motor	5.7×10^{-7}	$Kg.m^2$
R_a	Armature resistance	1.9	Ω
η_m	Motor efficiency	0.8	
J_{eq}	Equivalent moment of inertia at the load	0.0053	$Kg.m^2$
m	Mass of pendulum	0.05	kg
L	Length to pendulum's center of mass	0.1325	m
J_B	Equivalent moment of inertia at pendulum's center of mass	0.0004256	$Kg.m^2$
R	Rotating arm length	0.178	m

$$\begin{cases} \ddot{\alpha} = \dfrac{1}{f(u)}[b.\cos\alpha.(c.V_m - d.\dot{\theta} - b.\sin\alpha.\dot{\alpha}^2) + a.f.\sin\alpha] \\ \ddot{\theta} = \dfrac{1}{f(u)}[e.(c.V_m - d.\dot{\theta} - b.\sin\alpha.\dot{\alpha}^2) + b.\cos\alpha.f.\sin\alpha] \end{cases} \quad (3)$$

where:

$$\begin{cases} V_m : \text{motor input voltage} \\ f(u) = ae - b^2\cos^2\alpha \end{cases} \quad (4)$$

The above equation describing the dynamics of RIPS are highly nonlinear. The servo motor viscous friction has been included but the friction between the pendulum and the rotating arm has been neglected.

In order to design a linear regulator state feedback, we need to linearize the model. Equation (3) can be linearized by considering the equilibrium state of the system. If we assume α is small (i.e., when the pendulum is near its equilibrium point), we can linearize these equations. For small α, sinα~α and cosα~1. Also for small α, α_dot^2 is negligible, and we get the following linearized equation:

$$\begin{cases} \ddot{\alpha} = \dfrac{a.f}{h}\alpha - \dfrac{d.b}{h}\dot{\theta} + \dfrac{c.b}{h}V_m \\ \ddot{\theta} = \dfrac{b.f}{h}\alpha - \dfrac{d.e}{h}\dot{\theta} + \dfrac{c.e}{h}V_m \end{cases} \quad (5)$$

Where: $h = a.e - b^2$ (6)

The linearized model is used to design the stabilizing controller. We choose the state variable x(t) as:

$$x(t) = [\theta\ \alpha\ \dot{\theta}\ \dot{\alpha}]^T \quad (7)$$

From (5) we obtain the following linear state space model of the RIPS:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & bf/h & -ed/h & 0 \\ 0 & af/h & -bd/h & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ ce/h \\ cb/h \end{bmatrix} V_m \quad (8)$$

Substituting parameters from Table. 1 into (8), the linearized model of the RIPS results in:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 7.4255 & -35.7019 & 0 \\ 0 & 56.5797 & -32.3055 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 41.0410 \\ 37.1309 \end{bmatrix} V_m \quad (9)$$

III. Stabilizing Controller Design

The Linear Quadratic Regulation (LQR) [2][7][9][10] is used for the calculation of the optimal gain matrix K such that state feedback law:

$$u_{balance} = -Kx(t) \quad (10)$$

The minimizes of the cost function is defined as:

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt \quad (11)$$

The subject to the state dynamics is defined as:

$$\dot{x} = Ax + Bu \quad (12)$$

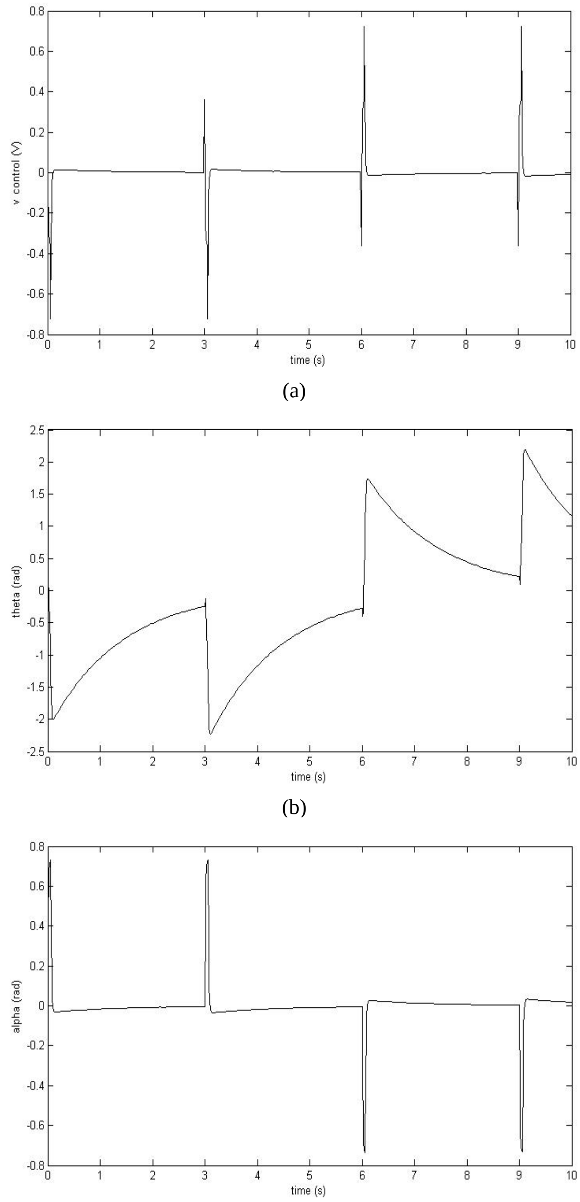
Using MATLAB, the LQR balancing controller for the RIPS is designed based on the required weighting matrices:

$$Q = diag([1,\ 1,\ 0,\ 0])\ and\ R = 1 \quad (13)$$

Resulting in the optimal gain:

$$K = [-1.0000,\ 19.1572,\ -2.0517,\ 2.6835] \quad (14)$$

Simulation results at Fig. 4 and experimental result at Fig 5a show the balance mode by LQR controller is used to stabilize and maintain the pendulum in the upright position, even when the disturbance (attaching a screw driver at the tip of pendulum) is applied to the arm position as simulation results at Fig. 4 and experimental result Fig. 5b. The robustness of the RIPS is successfully realized by the proposed LQR controller.



(a) (b) (c)

Fig 4. Simulation results using LQR control method
(a) output voltage with disturbance $\alpha_o = \alpha \pm \pi/36$ rad
(b) arm position with disturbance $\alpha_o = \alpha \pm \pi/36$ rad
(c) pendulum position with disturbance $\alpha_o = \alpha \pm \pi/36$ rad

IV. Swing up Controller Design

A. Energy swing up controller

The swinging up of a pendulum from the downwards position can also be accomplished by controlling the amount of energy [8] in the system. The energy in the pendulum system can be driven to a desired value through the use of feedback control. By adding in enough energy such that its value corresponds to the upright position, the pendulum can be swung up to its unstable equilibrium.

When the pendulum is close to the upright position, the stabilizing controller designed earlier can be triggered to catch the pendulum and balance it around the unstable equilibrium.

The system is defined such that the energy, E, is zero in upright position. The energy of the pendulum can be written as:

$$E = mgL\left[\frac{1}{2}\left(\frac{\dot{\alpha}}{\omega_o}\right)^2 + \cos\alpha - 1\right] \quad (15)$$

ω_o is the frequency of small oscillations around the downwards position:

$$\omega_o = \sqrt{\frac{mgL}{J_{eq}}} \qquad (16)$$

Equation (11) shows the energy in the pendulum is a function of the pendulum angle and the pendulum angular velocity. Note also that the energy corresponding to the pendulum in the downwards position is -2mgL. The goal of the control scheme is to add energy into the system until the system until the value corresponds to the pendulum in the upright position.

The control law implemented to achieve the desired energy is:

$$\ddot{\alpha} = sat_V(k(E - E_o))sign(\dot{\alpha}\cos\alpha) \qquad (17)$$

Where k is a design parameter and E_o is the desired energy level. The control output is the acceleration of the pivot which can be translated to a voltage input to the arm motor [1] by using equation (14):

$$\begin{cases} F = m\ddot{\alpha} = \dfrac{T}{r} \\ T = K_1 u - K_2\dot{\theta} \end{cases} \qquad (18)$$

Where T is output torque on the load from the motor.

E_o is a constant, we find it from experiment system, when the pendulum is within ±5º of the upright position and the angular velocity is slower than 2.5 radians per second.

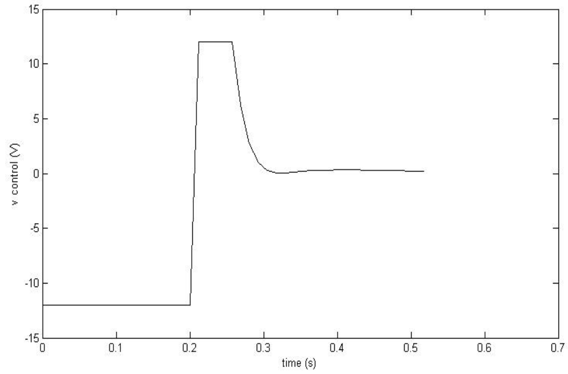
In our case the motor is strong and fast enough to swing up the pendulum in a single swing. We just have to set the desired velocity of the arm to its maximal value in one direction and when the pendulum reaches the horizontal position, we change the direction of arm's rotation. The desired velocity is maximal again. When the pendulum approaches the upright position, the LQR controller is engaged and the pendulum is stabilized.

Simulation results at Fig. 6 with k=-300 show the energy swing up controller is successful to swing up the pendulum in a single swing at 0.5618 second.

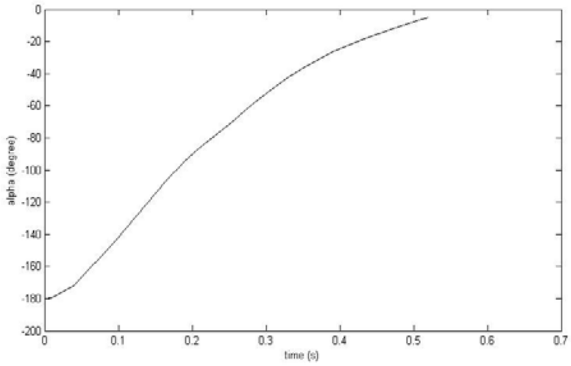
(a)

(b)

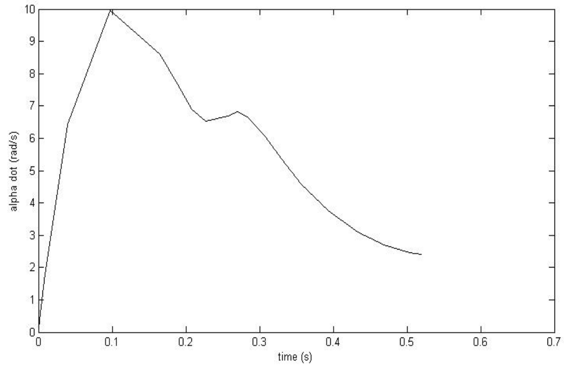
Fig 5. Experimental RIPS stabilizing controller using LQR
(a) without disturbance (b) with disturbance



(a)



(b)



(c)

Fig 6. Simulation results swing up using energy controller
(a) output voltage
(b) pendulum position
(c) pendulum velocity

B. *Heuristic swing up controller*

The heuristic controller [4][5][11] is a logic based control design that determines the direction and the moment in time the arm should move depending on the state of the system. A specific voltage gain is applied to the motor based on results from repeated experimentation.

This design restricts the arm movement between ±45º. When α and θ are zero, i.e. the arm and the pendulum are stationary; control voltage is maximum.

When α is increasing in one direction and velocity of α is zero, then control voltage is set in opposite direction.

And when $\alpha \geq \pm 90^o$ from the downwards position, control voltage is set zero.

The arm is swung at an appropriate frequency in either direction, the pendulum momentum increasing with each swing.

This way pendulum oscillates building up sufficient energy to reach near the desire value of π radian, which is vertical up position.

The voltage gain of this control scheme is determined by repeated experimentation. There is a direct correlation between the time it takes to swing the pendulum to its upright position and the magnitude of the voltage gain. A gain that is too high, though, may take the pendulum approach the upright position with too high velocity and, thus, the stabilizing controller will be unable to balance the pendulum. On the other hand, a gain too low may not provide enough energy to the pendulum so that it can not reach the upright position. Also, the reliability of the controller in performing the task varies depending on the gain selected. Thus, repeated experimentation is required to finely tune the gain so that the pendulum approaches the upright position with just the right amount of velocity and in a reasonable amount of time with a high success rate.

Real time results at Fig. 7 with voltage gain is 1.025 V show the heuristic swing up controller is successful to swing up the pendulum in about five swings at 2.6 second.

V. MODE SWITCH CONTROLLER DESIGN

Now that it is possible to get the pendulum close to upright position the stabilizing controller has to keep it there. To activated the stabilizing controller a switching strategy has to be implemented to switch from the swing up mode to balance mode. So this strategy has to say when the stabilizing controller can take over from the swing up controller.

It is almost impossible to switch when the pendulum is at exactly $\alpha=0^o$ and since the stabilizing controller will have some range as attraction as well it is smart to choose a range of $\phi=\pm 5^o$ from the vertical position to switch over from one to another.

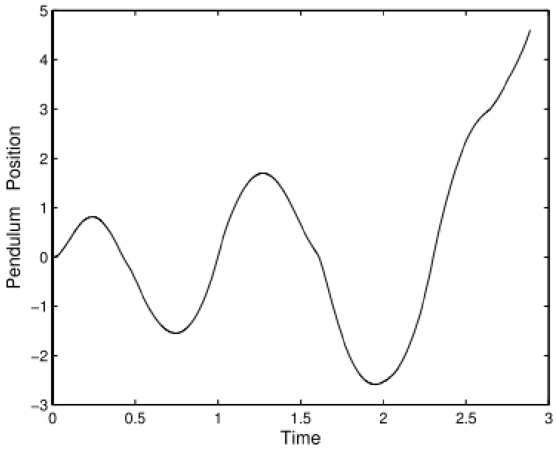


Fig 7. Real time result of swing up using heuristic controller

So if the pendulum position is within ±5º from the vertical, the program will switch controller. In case the stabilizing controller "looses" the pendulum the swing up controller will be active again. To prevent the controller from chattering when the pendulum position is close to ±5º from the vertical the switch out point is placed lower than the switch in point $\gamma=\pm 20^o$ as Fig 8.

At the hanging down position of pendulum, using the energy swing up controller, that helps pendulum swing up with a single swing. And when the pendulum move from upward balance area to unstable area, using the heuristic swing up controller will bring the pendulum return upward balance area fast and smoothly.

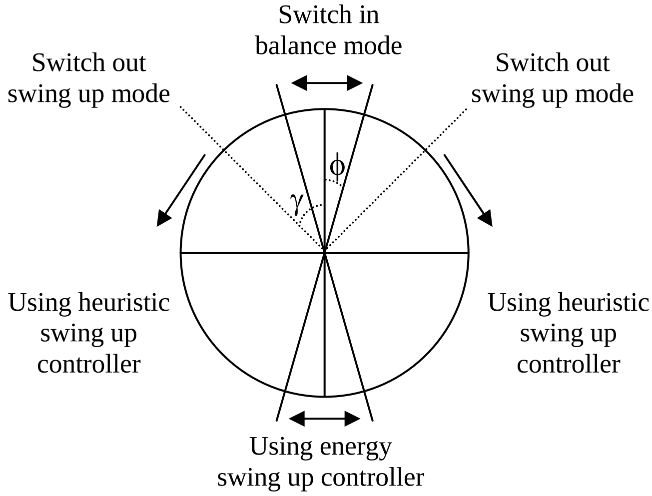


Fig 8. Switching areas for the switching strategy

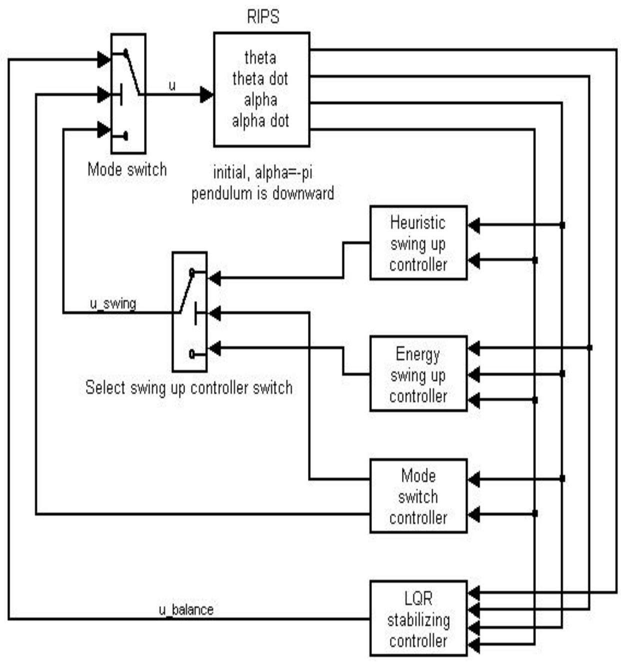


Fig 9. Implementation block diagram of RIPS

The switch that triggers the stabilizing controller in both case is activated when the pendulum is within ±5º of the upright position and the angular velocity is slower than 2.5 radians per second for stabilizing controller to catch the pendulum successfully. Implementation block diagram [10][12] of RIPS as Fig. 9.

VI. CONCLUSIONS

In this paper, two swing up control scheme has been implemented that will switch to a stabilizing controller when the pendulum is near the upright position in order to balance the pendulum. After that, the balance mode by LQR controller is used to stabilize and maintain the pendulum in the upright position, even when the disturbance is applied to the arm or the pendulum position. The robustness of the RIPS is successfully realized by the proposed LQR controller.

The main contribution of this paper is designed an hybrid swing up controller which is not only fast to swing the pendulum up at hanging down position but is also flexible to bring the pendulum return upright balance position when the pendulum move to unbalance zone.

The outcome of this paper can serve as a foundation for further studies of robot, serial double RIPS, and Segway self balance electric vehicle. The RIPS designed in this paper can be used for academic instruction in science education to sensitize, inspire, and entertain people.

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